Rabi Frequency and Oscillation in a Nonlinear Micro-ring Circuit

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Abstract

Rabi oscillation has been observed in many super-conducting devices, in which the quantum behavior between atom and electromagnetic field interaction is occurred. In this paper, we use an average two-level transient atom within the micro-ring resonator, which is consequently generated by the AlGaAsmaterial interaction with the electromagnetic field in time-domain. The simulation results obtained have shown that the coupling intensity is affected to the output intensity oscillation and also directly affected to the Rabi frequency. The probability to find atom in the excited state of the device with the reducing frequency at resonance is also calculated, in which the probabilities of 0.85 and 0.20 are obtained for resonance and non-resonance states, respectively. The Rabi frequency oscillation in the range of terahertz is obtained, which can be useful for atom antenna and sensing applications.

Keywords: Nonlinear optical device; Rabi frequency; Rabi oscillation; Nonlinear integrated optics

1. Introduction

A small scale optical device known as a microring resonator has been widely studied and investigatedin many applications [1-5], where one of them has shown the interesting aspect [6], in whichthe use of nonlinear microring resonator(circuit) for optical signal processing with the advantages of small and low power consumption has been reported [7]. Moreover, the use of a nonlinear microring resonator has shown the broad potential of applications [8-12]. Such devices can be designed in the forms of add-drop filter, all-optical switching, modulators and optical logic gates for various applications, which can be used and embedded on the modern digital instruments such as mobile telephone handset, tablet, ipad and lap top computer.

Rabi oscillation is recognized as the basic property that can be used to describe the interaction between atom and the electromagnetic field, which is the dynamical system of transient atom from ground state to another state with the frequency known as Rabi frequency. In principle, the interaction between the electromagnetic fields within themedium material is a small-distance-scale with short-time-scale interaction, particularly, in the resonance regime and the associated device structures requires quantum mechanical description of the electronic state available in the medium. This system can be created in different ways, one of which is adding the interaction Hamiltonian term in the Hamiltonian system and consider the solutions that are time-dependent functions or frequency-dependent functions depending on the Fourier transformation, where finally, Rabi frequency and oscillation can be determined.

In this paper, the Rabi oscillation of particle (photon) propagates within a microring resonator is described by the set of Hamiltonian, which is in the form of field and its interaction of transient photon from ground state to the excited state, which is formed by the average two level atoms. It is related to the Rabi frequency in time-domain for finding the probability of atom at the excited state at each period. In simulation, a system is applied to the nonlinear microring circuit, which is the Alumilium Gallium Arsenide (AlGaAs) [13-16],where the practical device parameters are used in the simulation. It is notthe silicon device. Therefore, AlGaAs has the nonlinear optical properties at the various wavelengths and its ultrafast nonlinearity higher than silicon, which is suitable for high-speed passive-waveguide usage. From the obtained simulation results, we have discussed the potential of using Rabi frequency and oscillation within a nonlinear microring circuit for atom antenna and sensing applications. In addition, the new design for high capacity Rabi frequency and oscillation using a PANDA ring circuit is also discussed. This is required the theoretical description and useful for large scale devices, which is useful for atom network and cellular automata applications.

2. Theoretical Background

In this study, two-level atom (photon) is used to manipulate light as a particle that propagates in a nonlinear microring resonator, wherethe ground and excited states of interaction between atom and electromagnetic field are given by $|\psi_g\rangle$ and $|\psi_e\rangle$ with energy eigenvalue E_g and E_e, respectively.

The energy different is then related to the transition energy of atom, $\hbar\omega_0 = E_e - E_g$, with atomic transition angular frequency(ω_0). The Hamiltonian of the two-level system is given by [17-18]

$$H = H_A + H_F + H_{\text{int}} . \quad (1)$$

Where H_A and H_F are the unperturbed Hamiltonian operators of atom and the field, respectively, which can be written as

$$H_A = P^2 / 2M + \hbar\omega_0\pi^\dagger\pi , \quad (2)$$

$$H_F = \hbar\omega a^\dagger a . \quad (3)$$

Where P is the center-of-mass momentum operator, M is the mass of the atom, π and $\pi^\dagger$ are the atomic raising and lowering operators, respectively. a and $a^\dagger$ are the creation and annihilation operators, respectively, with a photon energy $\hbar\omega$. The interaction Hamiltonian is given by $H_{\text{int}} = -d \bullet \tilde{E}(t)$, which is described the coupling of the electromagnetic field to atom and related to the electric dipole moment operator, d. At this point we consider a spatially uniform which is well known as the dipole approximation. The atomic behaviors are described by the general state that can be expressed as a linear combination of them. Here, the general state is the evolution of time and perturbed by electromagnetic field, where each component evolves with its characteristic exponential factor, which is given by

$$\left|\psi(t)\right\rangle = c_g(t)e^{-iE_g t/\hbar}\left|\psi_g\right\rangle + c_e(t)e^{-iE_e t/\hbar}\left|\psi_e\right\rangle , (4)$$

Where $|c_g(t)|^2$ and $|c_e(t)|^2$ are the probabilities of the states $\left|\psi_g\right\rangle$ and $\left|\psi_e\right\rangle$, respectively. The states $\left|\psi_g\right\rangle$ and $\left|\psi_e\right\rangle$ are the position-space wave functions, in which the orthonormal state is $\left\langle\psi_g\middle|\psi_e\right\rangle = \delta_{ge}$. The dynamics of $c_g(t)$ and $c_e(t)$ can be found by demanding that the state $\left|\psi(t)\right\rangle$ satisfies the time-dependent Schrödinger equation with the rotating wave approximation, where the rapidly oscillating term, $|\omega - \omega_0| \ll \omega$ is neglected, then the final form is written as

$$\dot{c}_g = i\Omega^*(\mathrm{r})c_e e^{i\Delta t} , \quad (5a)$$

$$\dot{c}_e = i\Omega(\mathrm{r})c_g e^{-i\Delta t} . \quad (5b)$$

Where $\Delta \equiv \omega - \omega_0$ is the detuning, $\Omega(\mathrm{r})$ is the Rabi frequency which represents the frequency of oscillation for a given atomic transition in a given light field, which is defined by $\Omega(\mathrm{r}) = d_{eg} \bullet E(\mathbf{r}) / \hbar$. The equation (5) is the well-known Rabi oscillationthat occurs between the ground and excited states of the driven two-level system. From the trial solution, $c_e(t) = e^{i\lambda t}$, and considering the initial condition $c_g(0) = 1$ and $c_e(0) = 0$. The probability to find atom in state $\left|\psi_e\right\rangle$ is given by

$$P_e(t) = |c_e(t)|^2 = (4I / \Omega_R^2\hbar^2)\sin^2(\Omega_R t / 2) , (6)$$

Where Ω_R is the total Rabi frequency, $\Omega_R = \sqrt{\Delta^2 + 4\Omega^2}$, and we also set $E(\mathbf{r}) \approx E(z) = \varpi A(z)e^{i\beta_0 z}$, where $\varpi = (\mu_0 / \varepsilon_0)^{1/4}(2n_0)^{1/2}$. μ_0 and ε_0 are the permeability and permittivity in vacuum, respectively. n_0 is the linear refractive index, $\beta_0 = n_0 k$ is the propagation constant with $k = \omega / c$

, c is the speed of light and $A(z)$ is the complex amplitude ($|A|^2$ is the intensity, I).

A schematic diagram of a nonlinear microring resonator with the notation used is as shown in Fig. 1. Let us consider, a continuous wave (CW) signal at an angular frequency, ω ,propagating in a straight AlGaAs waveguide coupled laterally to a ring of radius R . We consider the evolution of electric field associated with this optical wave is described by a nonlinear differential equation. For a practically case in which the impact of two-photon absorption(TPA) and free-carrier effecton the attenuation of optical wave are relatively small compared with other loss sources [19-20]

$$\partial A/\partial Z = -\alpha A/2 + i\gamma |A|^2 A, \qquad (7)$$

The parameters entering Eq. (7) are as follows: α and $\gamma = kn_2$ are the linear loss and the Kerr effect, where k and n_2 are the wave number and nonlinear refractive index. The solution of equation (7) is given by $A(z)=\sqrt{I(z)}\exp[i\phi(z)]$, therefore the required result in term of intensity is expressed by

$$I = I_0 e^{-\alpha z}, \phi = \phi_0 + \gamma I_0 L_{\text{eff}}, L_{\text{eff}} = \left[e^{-\alpha z} - 1\right]/\alpha , (8)$$

Where I_0 and ϕ_0 are the values of intensity and phase at z = 0, and L_{eff} is the effective length. Then the electric fields on both sides of the point coupler satisfy the following relations

$$E_4 = rE_1 + itE_3,\ E_2 = rE_3 + itE_1. \qquad (9)$$

Where $t=\sqrt{1-r^2}$ and r^2 is the fraction of power remaining in the straight waveguide after the coupler. We can express the intensity $I_4 = |E_4|^2/\varpi^2$ in terms of $I_0 = |E_2|^2/\varpi^2$ and their conservation law of energy as

$$I_4(I_0) = \left[r^2 I_0 + I(2\pi R) - 2r\sqrt{I_0 I(2\pi R)}\cos\Delta\phi\right]/(1-r^2)$$
(10)

$$I_1(I_0) = I_0 - I(2\pi R) + I_4(I_0)\ . \qquad (11)$$

Where $\Delta\phi = 2\pi\beta_0 R + \phi(2\pi R) + \phi_0$ is the phase shift acquired during a round trip in amicroringcircuit.

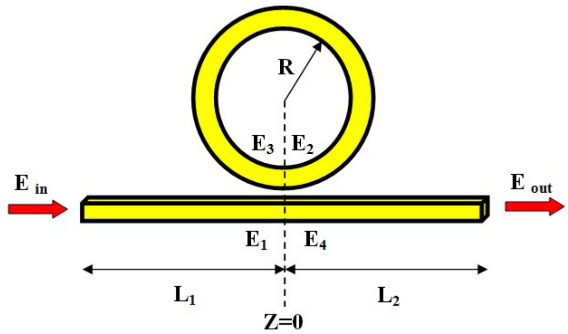
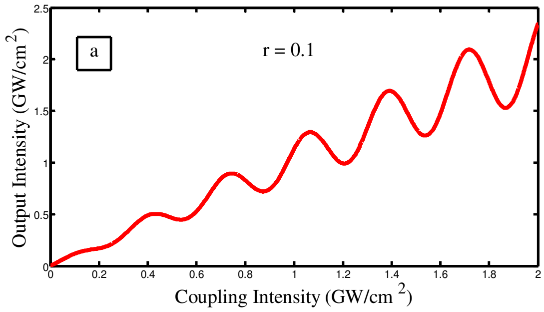


Figure 1. A schematic of AlGaAs ring resonator and the notation details, where E_i: Optical fields, R: Ring radius, L: Device length, Z: Ring center

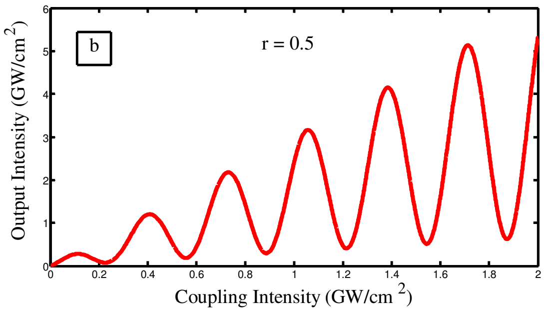
3. Results and Discussions

In simulation, the above calculation was carried out for an AlGaAs ring resonator with a radius of R = 5 μm, with wavelength λ = 1,550 nm and length of L_1 = L_2 = 10 μm, n_0 = 3.34, α = 5 $d\beta$/cm and n_2 = 1.5×10^{-4} cm^2/GW. The numerical results are calculated by using the commercial MATLAB software. The results obtained are compared with the evolution of coupling intensity at z = 0 versus output intensity by varying the coupling coefficient r in absence of TPA and free-carrier effect as illustrated in Fig. 2. The optical oscillation disappears when the strength AlGaAs waveguide performed by theAlGaAs ring resonator, where the oscillation is increased and higher amplitude obtained, when the coupling coefficient is increased.The oscillation is related

to the Rabi frequency, i.e. the atomic transition level frequency, with is shown in Fig. 3a. In fact, the Rabi frequency is the complex value, however, the real part is firstly calculated.To increase the coupling intensity, with the same coupling coefficient *r*, the absolute Rabi frequency output intensity is increased relatively large, while the absolute Rabi frequency at the output port depends on the output intensity as shown in Fig. 3b. The probability of finding the excited photons can be calculated by using the Rabi oscillation system, where is given by the details in Eq. (6), with the coupling coefficient *r* = 0.1. In Fig. 4, we see that the probability of the excited state at output port is 0.8417, which is increased within the resonance regime $\Delta = 0$. However, if the system is at off resonance regime $\Delta = 0.5\Omega$, the probability at the excited state is 0.1975, which is reduced(decreased). When the system is considered at the resonance and off resonance conditions, the oscillation time of the off resonance is within the range of 0 – 0.5 ps, where the oscillation time of resonance is about 0.246 ps, while the off resonance is about 0.086 ps. This is shown that trend of probability's time is increased steadily.



(a)



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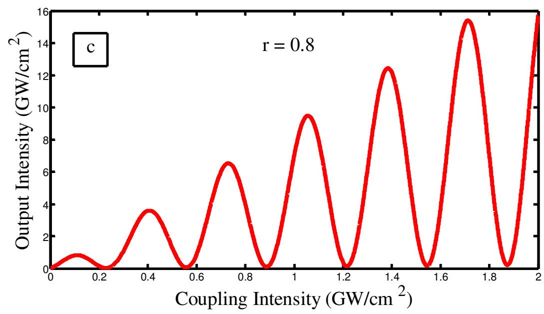
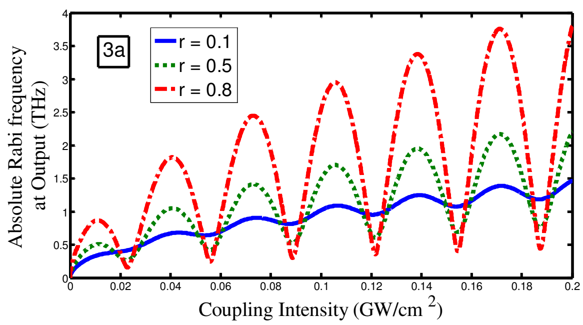
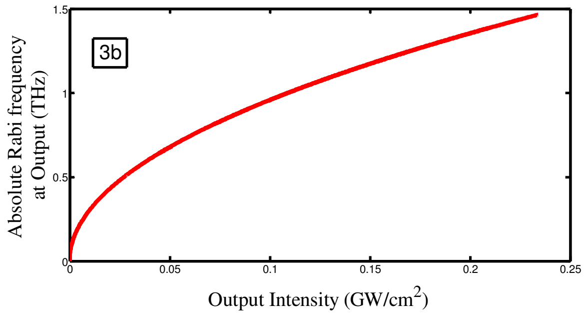


(c)

Figure 2. The effect of coupling coefficients of the coupling-output characteristics,where (a) r = 0.1, (b) r = 0.5, (c) r = 0.8.



(3a)



(3b)

Figure 3. Graph of absolute Rabi frequency, where (3a) comparison between the difference coupling coefficients when the system performed by coupling intensity; (3b) at output port.

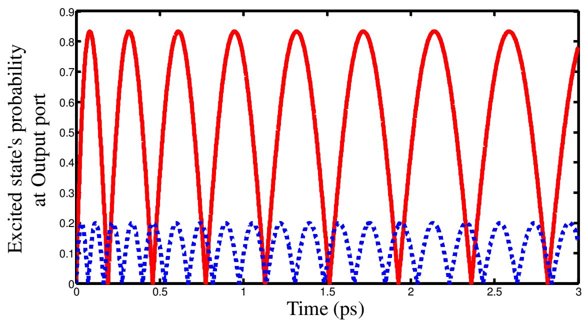


Figure 4. Result of Rabi oscillations at output port, with the coupling coefficient r = 0.1: when Resonance, $\Delta = 0$ (solid curve) and Off resonance , $\Delta = 0.5\Omega$ (dashed curve).

4. Conclusion

We have proposed the use of microring resonator circuit for photon oscillation investigation, where theAlGaAs ring resonator is formed by the strength AlGaAs waveguide. Photon in such device is oscillated depended on the coupling coefficient which affects to the Rabi frequency at coupling range, while the optical oscillations have not affects to the Rabi frequency out put intensity. The probability of finding atom in the excited state has high amplitude when it is in the resonance. Theoscillation time of probability with increasing frequency over the range of the resonance.This is a firtly calculated result of Rabi oscillation, which can be useful for further applications, where more complicated circuits such as add-drop filter and PANDA ring resonator and casecade system, in which the greater Rabi frequency range can be obtained with greater intensity, which is useful for the applications such as quantum information,atom antenna and cellular automata, atom computing and atom sensors.

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